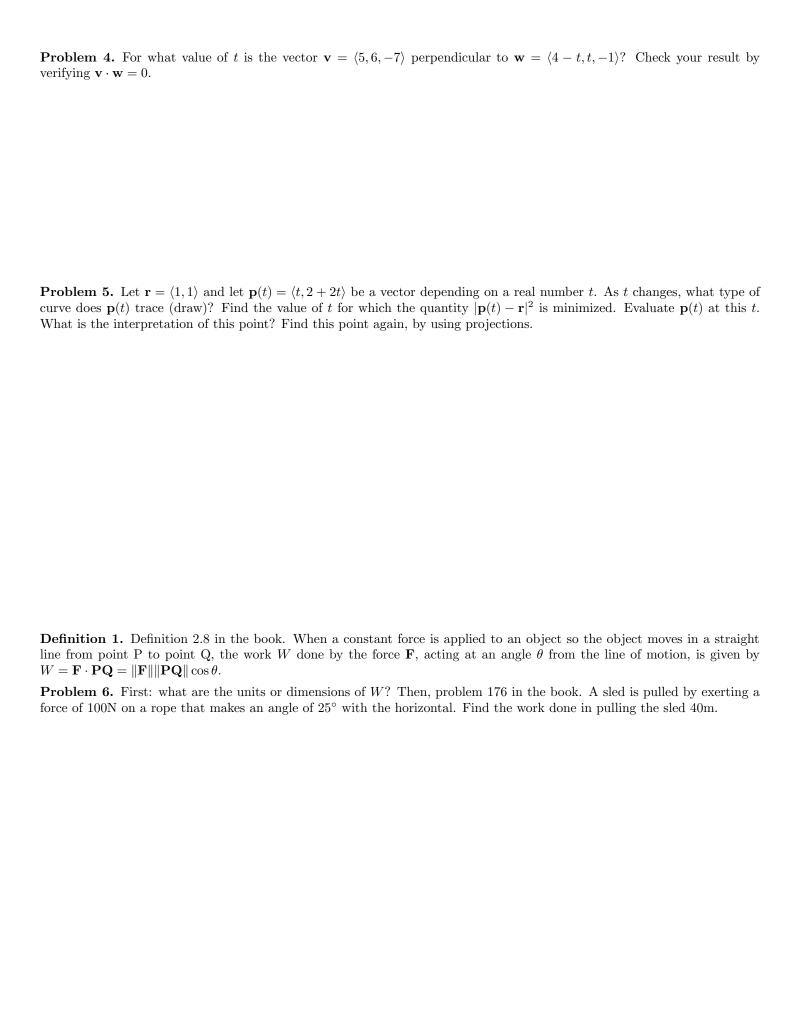
MATH 233 Recitation All questions

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	plem 1. Let $\mathbf{v} = \langle 1, 0, 3 \rangle$ and $\mathbf{w} = \langle 0, -2, 4 \rangle$. Compute $\mathbf{v} \cdot \mathbf{w}$.
(b)	What is the angle between ${\bf v}$ and ${\bf w}$?
(c)	Let $\mathbf{z} = \mathbf{w} - \frac{\mathbf{v} \cdot \mathbf{w}}{ \mathbf{v} ^2} \mathbf{v}$. Compute the components of \mathbf{z} .
(d)	Compute $\mathbf{z} \cdot \mathbf{v}$ and $\mathbf{z} \cdot \mathbf{w}$.
(e)	Compute $ \mathbf{z} ^2$.

(f) What is the angle between ${\bf z}$ and ${\bf v}$? What is the angle between ${\bf z}$ and ${\bf w}$?





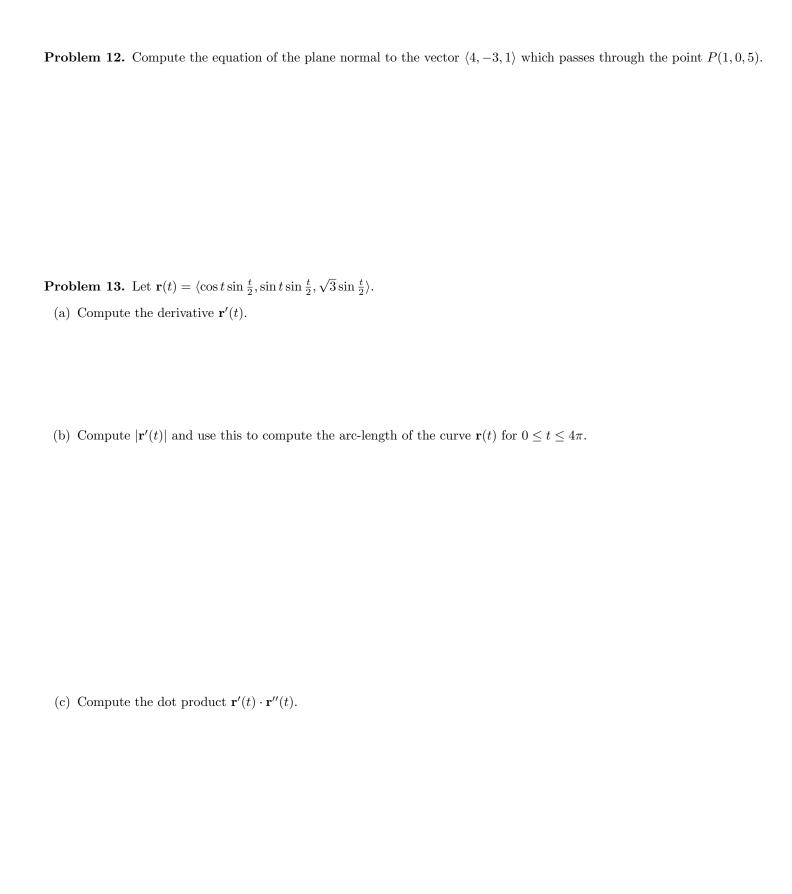
Problem 7. Determine if the following claims are true or false. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be vectors in \mathbb{R}^3 (three dimensions). If false, give a simple example that fails.

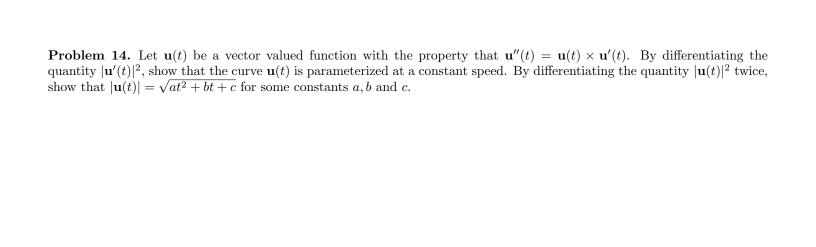
- (a) $\mathbf{u} \times \mathbf{u} = \mathbf{0}$.
- (b) $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.
- (c) If $\mathbf{u}, \mathbf{v} \neq \mathbf{0}$ and $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ then $\mathbf{u} = \mathbf{v}$.
- (d) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$.
- (e) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \mathbf{u} \cdot (\mathbf{w} \times \mathbf{v})$
- (f) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \mathbf{v} \cdot (\mathbf{w} \times \mathbf{u}) = \mathbf{w} \cdot (\mathbf{u} \times \mathbf{v})$
- (g) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$

Problem 8. Compute $\mathbf{u} \times \mathbf{v}$ where $\mathbf{u} = \langle -1, 0, e^t \rangle$, $\mathbf{v} = \langle 1, e^{-t}, 0 \rangle$.

Problem 9. Compute $\mathbf{w} \cdot (\mathbf{v} \times \mathbf{u})$ and $\mathbf{u} \cdot (\mathbf{w} \times \mathbf{v})$ where $\mathbf{u} = \langle 4, 2, -1 \rangle$, $\mathbf{v} = \langle 2, 5, -3 \rangle$, and $\mathbf{w} = \langle 9, 5, -10 \rangle$.







Problem 15. Sketch the level curves of the functions $f(x,y) = x^2 + y^2$, and g(x,y) = xy - x.

Problem 16. Compute the arc-length of the curve $\mathbf{r}(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle$ from $0 \le t \le 3$.





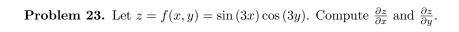
Problem 22. Compute the following limits or show that they do not exist. If they do not exist, check whether the limit along different curves exists and differs by the curve.

(a)
$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

(b)
$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2}$$

(c)
$$\lim_{(x,y,z)\to(0,0,0)} \frac{x^2-y^2-z^2}{x^2+y^2-z^2}$$

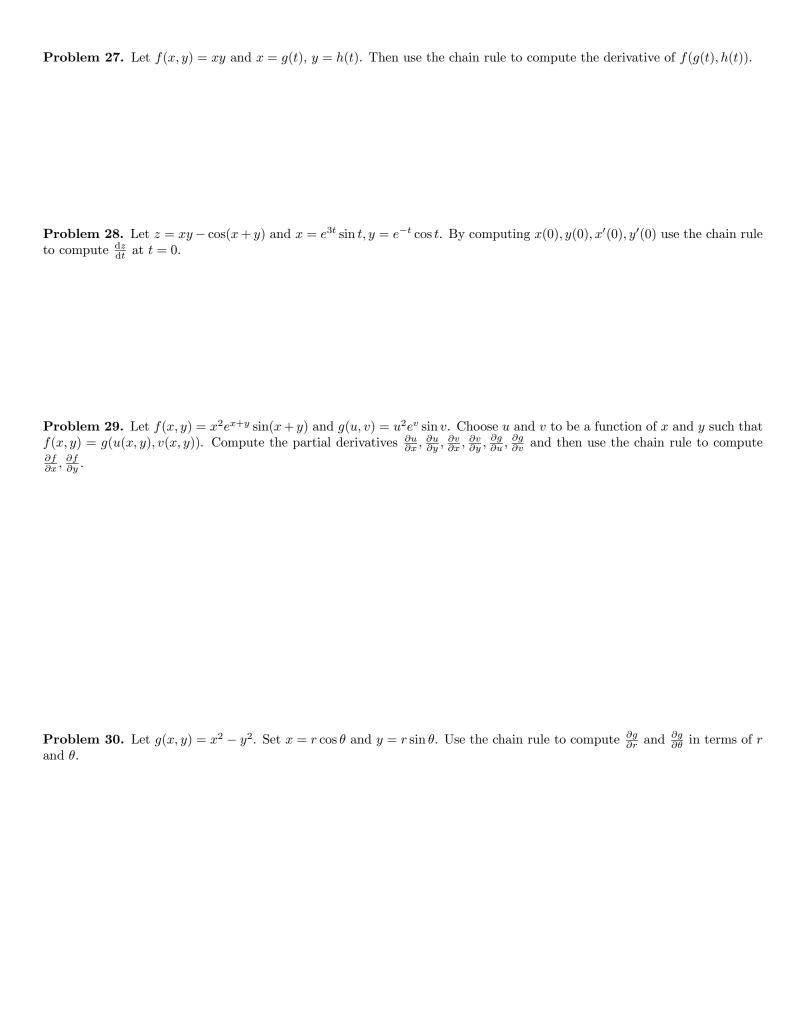
(d)
$$\lim_{(x,y)\to(0,0)} \ln(x+y)$$



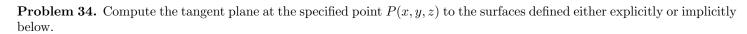
Problem 24. Let $z = f(x, y) = \exp^{xy} \sin(x) \cos(y)$. Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Problem 25. Show that $z = e^{-t} \cos\left(\frac{x}{c}\right)$ satisfies the heat equation $\frac{\partial z}{\partial t} = c^2 \frac{\partial^2 z}{\partial x^2}$ (compute the derivatives and check if the equality is true)

Problem 26. Compute the Hessian (second derivatives), then find and classify the critical point for the function $z = x^2 - xy + y^2 - 3x - 2y$.







(a)
$$z = -9x^2 - 3y^2$$
, $P(2, 1, -39)$

(b)
$$xy + yz + zx = 11, P(1, 2, 3)$$

(c)
$$-8x - 3y - 7z = -19$$
, $P(1, -1, 2)$

Problem 35. Use the multi-variable chain rule to compute $\frac{df}{dt}$ where $f(x,y)=x^2+y^2$ and $x(t)=t, y(t)=t^2$. Repeat this calculation directly by substituting x(t), y(t) and differentiating f(x(t), y(t)). Draw a schematic of the setup and interpret the derivative.

$$\int_0^1 \int_0^2 x^2 y - y^3 x \, \mathrm{d}x \, \mathrm{d}y.$$

Problem 37. Let R be the region $\{(x,y): x>0, 2x< y< x+1\}$. Draw a picture of this region then integrate

$$\iint_R xy + y \, \mathrm{d}x \, \mathrm{d}y$$

Problem 38. Let D be the region bounded between the curves $x = y^2$ and x + y = 2. Set up an integral to compute the area of this region, and then compute

$$\iint_D x \mathrm{d}x \mathrm{d}y$$

.

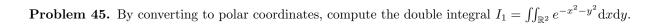
Problem 39. Let $h(x,y) = x^2 + y^2 + 1$ and g(x,y) = 1 - x - y. Compute the volume of the region bounded above by z = h(x,y), below by z = g(x,y) and on the sides by x = 0, x = 1, y = 0 and y = 1.

Problem 40. Compute the iterated integral

$$\int_0^{\pi/3} \int_0^x \sin(x+y) \, \mathrm{d}y \, \mathrm{d}x$$

Problem 41. Let $z = xy - \cos(x + y)$ and $x = e^{3t} \sin t$, $y = e^{-t} \cos t$. Compute x(0), y(0), x'(0), y'(0) and apply the chain rule to evaluate $\frac{dz}{dt}$ at t = 0.





Use the fact that $e^{-x^2-y^2}=e^{-x^2}e^{-y^2}$ to relate I_1 and the Gaussian integral $I_2=\int_{-\infty}^{\infty}e^{-x^2}\mathrm{d}x$ and thus compute I_2 .

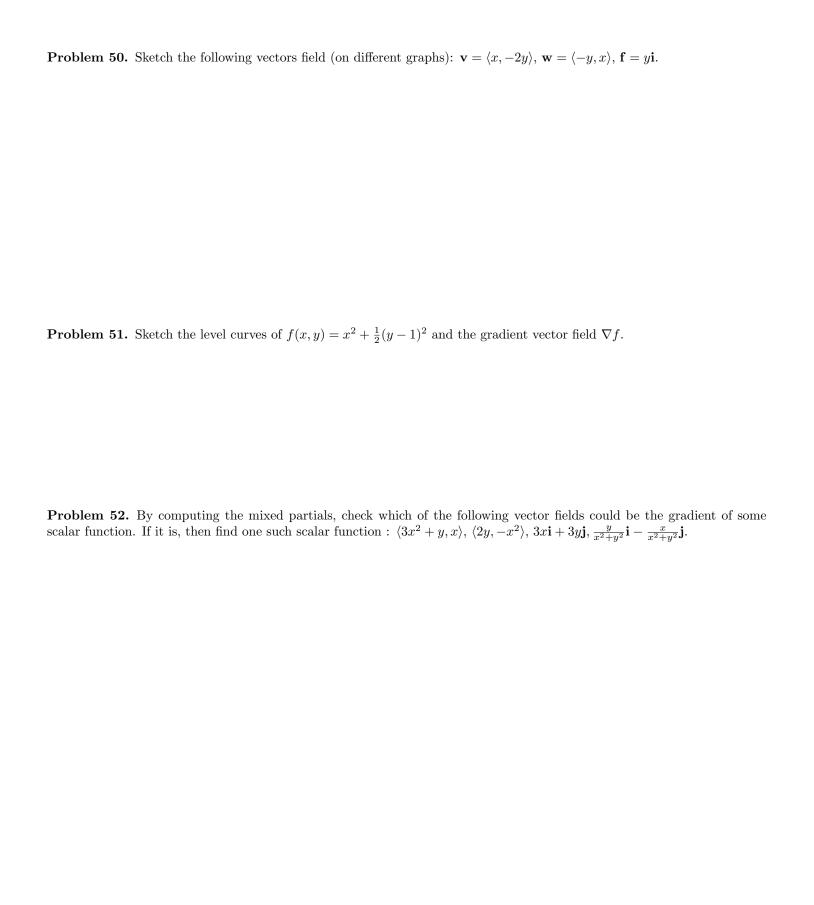
Use the same strategy to compute $\int_{-\infty}^{\infty} e^{-sx^2} dx$ for any s > 0.

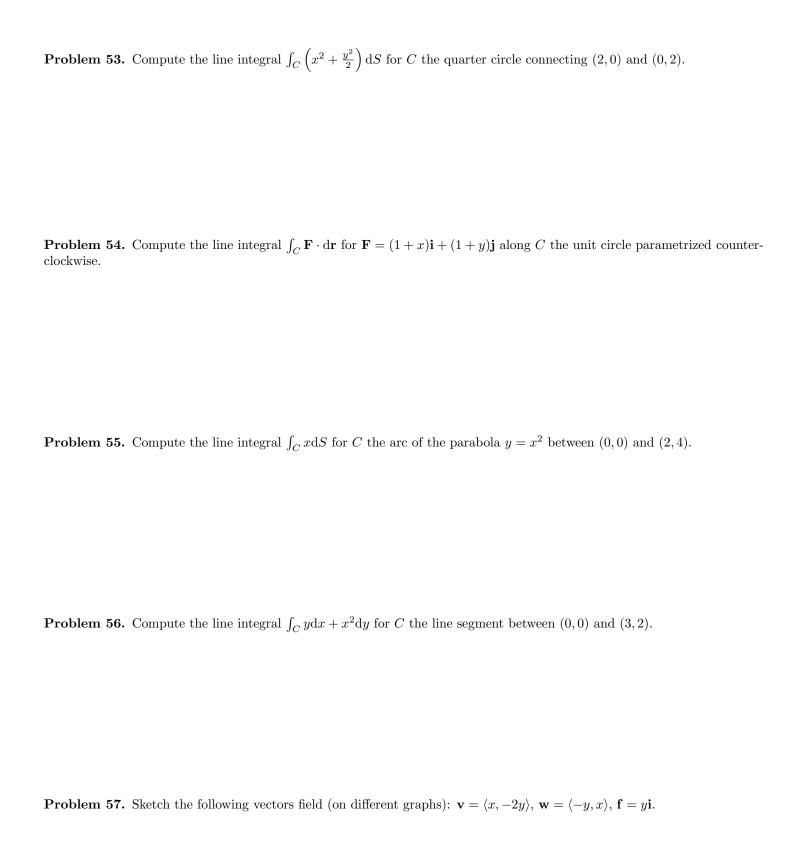
Problem 46. Let C be the right cone with base centered at the origin of radius 5, and of height 4. Using cylindrical polar coordinates compute

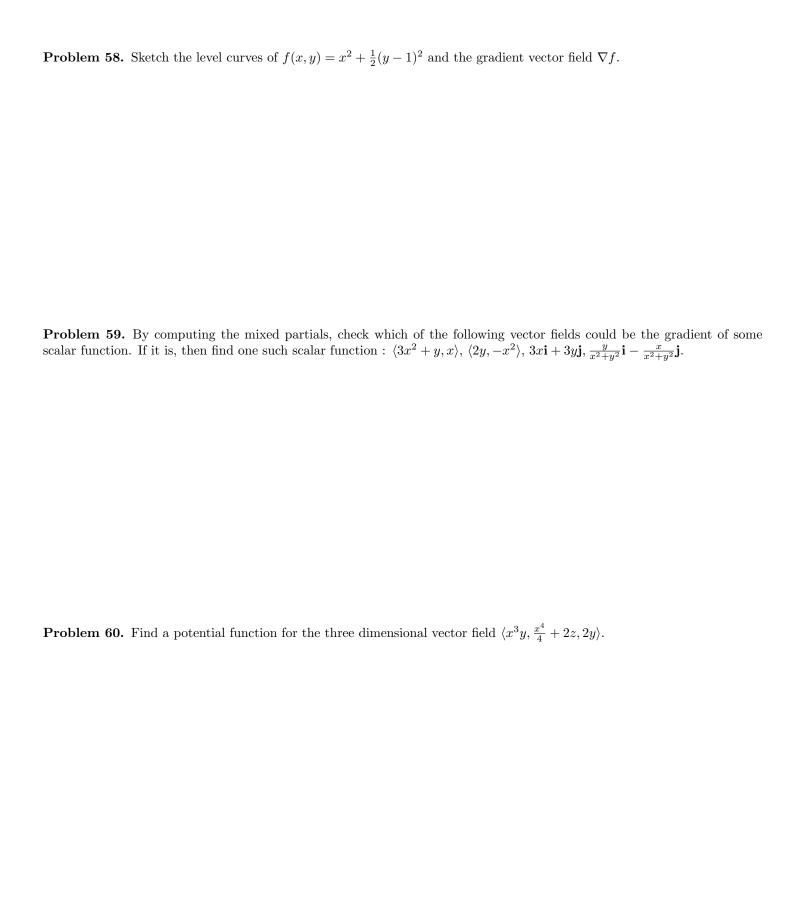
 $\iiint_C z \, \mathrm{d}x \mathrm{d}y \mathrm{d}z.$

Do you know what this quantity represents physically?

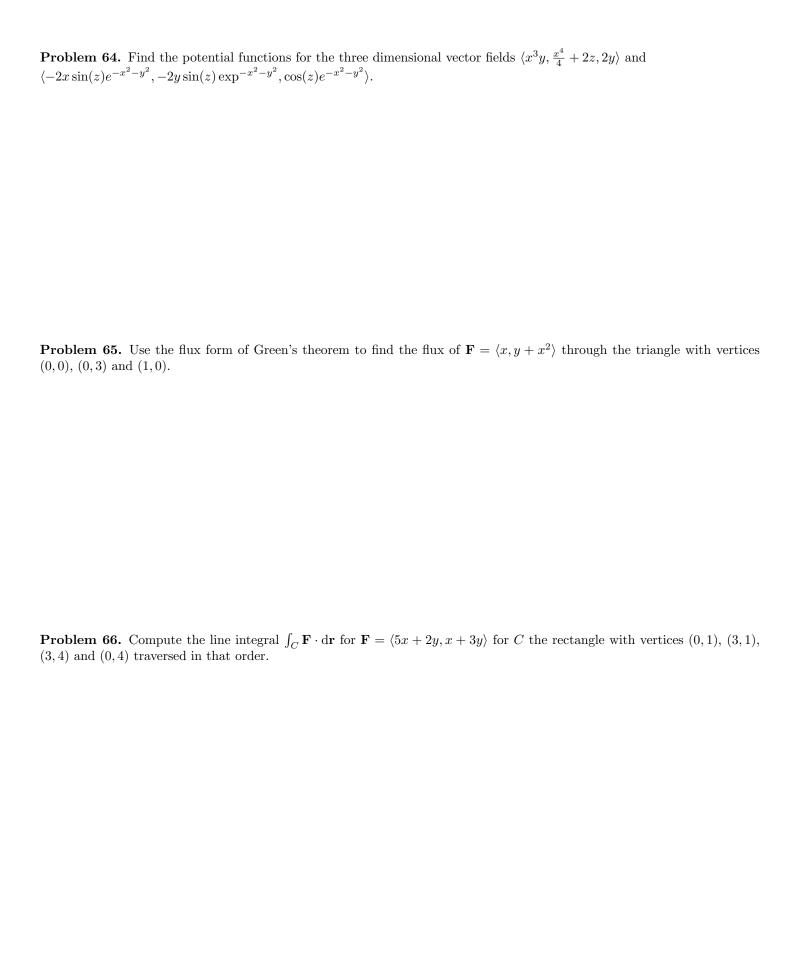


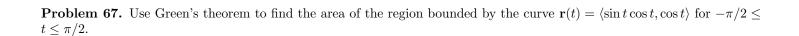












Problem 68. Compute the divergence of vector fields

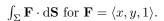
(a)
$$\mathbf{F} = \langle x^2 + y^2, y^2 + z^2, x^2 + y^2 \rangle$$

(b)
$$\mathbf{G} = \langle xe^x, ye^y, ze^z \rangle$$

(c)
$$\mathbf{H} = \langle x^2 y, y^2 + x \rangle$$
.

Problem 69. Use the curl to check if the vector field $\mathbf{v} = \langle yz, xz, xy \rangle$ is conservative.

Problem 70. Describe the surface Σ given by the parametric formula $\mathbf{r}(u,v) = \langle u \cos v, u \sin v, u^2 \rangle$ for $0 \leq v < 2\pi$ and $0 \leq u \leq 2$. Compute the partials \mathbf{r}_u and \mathbf{r}_v and then their cross product $\mathbf{r}_u \times \mathbf{r}_v$. Use this to to compute the surface integral



Compute also the surface integral $\int_{\Sigma} \sqrt{x^2 + y^2} dS$.

Problem 71. Compute the curl of the vector fields

(a)
$$\mathbf{R} = \langle x + y, -x, 3y - z \rangle$$

(b)
$$\mathbf{S} = \langle x^3 + y, -y + xz, x^2 \rangle$$
.

(c) Now compute the divergence of the curl $\nabla \cdot (\nabla \times \mathbf{S})$.

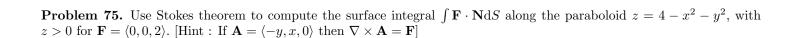
Problem 72. Using geometrical reasoning (without parameterizing the surface and doing the integral explicitly) compute flux $\int_{\Sigma} \mathbf{F} \cdot d\mathbf{S}$ for $F = \langle 3x, 3y, 3z \rangle$ and Σ the sphere of radius 2 centered on the origin.

Problem 73. Compute the curl of the vector field

(a) $\mathbf{S} = \langle x^3 + y, -y + xz, x^2 \rangle$.

(b) Now compute the divergence of the curl $\nabla \cdot (\nabla \times \mathbf{S}).$

Problem 74. Find the area of the surface given by the equation $x^2 + y^2 = z^6$ with $0 \le z \le 1$.



Problem 76. Let S be the quarter cylinder determined by the equation $x^2 + y^2 = 4$ and inequalities x > 0, y > 0, -1 < z < 1. Let $\mathbf{F} = \langle x, y, z^2 \rangle$ and compute the flux integral

$$\int \mathbf{F} \cdot d\mathbf{S}$$

.

Problem 77. Use a surface integral to find the height of the center of mass of a hemispherical shell of unit radius centered at the origin with z > 0. [Hint: Compute $\int z \, dS$ and divide by the surface area]

Problem 78. Use the divergence theorem to find the flux $\int \mathbf{F} \cdot \mathbf{N} dS$ flowing out of the rectangular box $0 \le x \le 3$, $1 \le y \le 2$, $0 \le z \le 1$ for the vector field $\mathbf{F} = \langle x^2 + y, x + y, z^3 + 3z \rangle$.